Evolutionary Algorithm for Interplanetary Trajectories Optimization

L. Casalino, M. Rosa Sentinella

Dipartimento di Energetica
Politecnico di Torino - Italy

Evolutionary Algorithms

- heuristic optimization methods
- randomly initialized population of individuals representing candidate solutions
- “fitness” assigned to each individual based on function to be optimized
- set of rules and parameters to force evolution toward improved fitness (i.e., performance)

Evolutionary Algorithms: Pros

- no assumptions about underlying problem: wide applicability
- global search capability
- no gradient evaluation required (discontinuous variables)
- no tentative solutions required
- well suited for problems with finite (possibly small) number of variables

Evolutionary Algorithms: Cons

- scarce accuracy ?
  not always !
- heuristic method: no certainty of finding correct solution (sometimes stuck on local optima)

Application to Interplanetary Trajectories

- patched-conic approximation
- solution defined by finite set of variables
  - relevant dates (departure, arrival, planetary encounters, impulses)
  - position of deep-space impulses
- EA individuals characterized by real values of the optimization variables
- fitness defined as inverse of mission ΔV (alternatively, as final mass or payload)

Genetic Algorithm

- based on selection and reproduction (and mutation)
- selection picks individuals (parents) for reproduction: tournament selection preferred
- crossover creates a new individuals (children): Deb's crossover with variable η adopted
  $y_1 = 0.5[(x_1 + x_2) - \bar{\beta}|x_2 - x_1|]$
  $y_2 = 0.5[(x_1 + x_2) + \bar{\beta}|x_2 - x_1|]$
- mutation: random change in a variable value (with small probability)
- elitism: best individuals preserved and passed to the following generation

Differential Evolution

- new individuals generated by adding the weighted difference between two (or more) population vectors to a third one

DE/best/1	$y = x_{best} + F(x_1 - x_2)$
DE/rand/1	$y = x_1 + F(x_2 - x_3)$
DE/rtb/2	$y = x_1 + F(x_{best} + x_2 - x_3)$
DE/best/2	$y = x_{best} + F(x_1 + x_2 - x_3 - x_4)$
DE/rand/2	$y = x_5 + F(x_1 + x_2 - x_3 - x_4)$
DE/rtb/1	$y = x_1 + C(x_{best} - x_1) + F(x_2 - x_3)$

- additional parameters to control which individuals are inserted into the new population

Particle Swarm Optimization

- individuals ("particles") possess velocity and fly in the search space
- cognitive and social accelerations (according to personal and global best) change velocity to direct the particle toward the optimum

$$v = v + c_1 \cdot k_1 \cdot (x_{P_{best}} - x) + c_2 \cdot k_2 \cdot (x_{G_{best}} - x)$$

$$y = x + v$$

- Trelea type 2 strategy is adopted

Improvements and Cooperative Approach

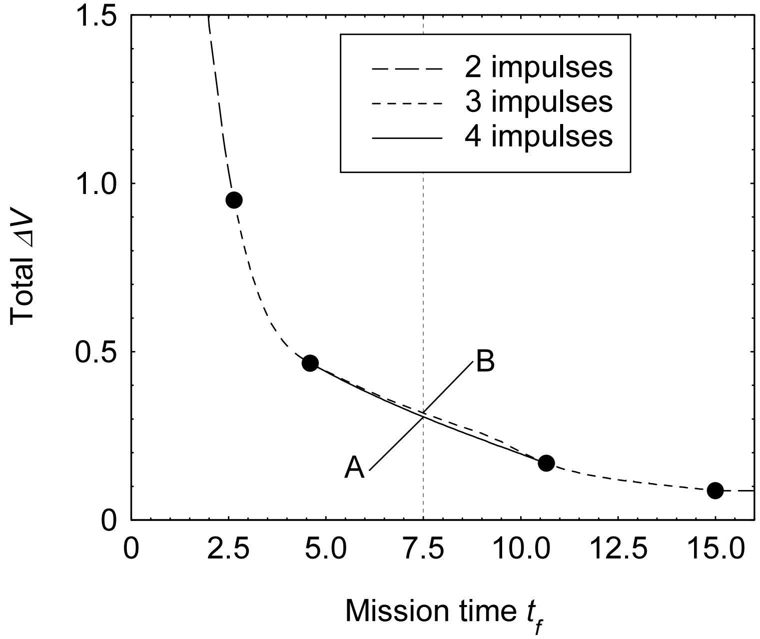
- extended random's initialization
- mass mutation
- parallel use of GA, DE and PSO
- best individuals copied to other populations (clonation) at prescribed intervals

Benefits of Cooperative Approach

- simple algorithms may converge only with particular sets of the parameters that control the evolution
- suitable settings may differ depending on the problem and are not known a priori
- cooperative approach improves chances of convergence to the global optimum
- increased speed when an algorithm with good exploration capabilities but slow convergence is coupled to one with opposite characteristics

Comparison of Basic and Hybrid Algorithms (1)

- planar circular rendezvous problem
- $r_i = 1$, $r_f = 1.2$, $\Delta\theta = 180^\circ$
- up to 4 impulses, 6 variables
- test at $t_f = 7.5$



Comparison of Basic and Hybrid Algorithms (2)

- hybrid algorithms with optimal settings

algorithm	ϵ	N_{min}	N_{mean}	S_N
DE	1.00	37500	183463	13254
GA	1.00	73198	139138	10577
PSO	0.30	9022	196702	25690
GA + DE	1.00	82338	122654	19067
GA + PSO	1.00	72793	154396	12423
DE + PSO	1.00	28069	80188	14623
GA + DE + PSO	1.00	72097	135516	13368

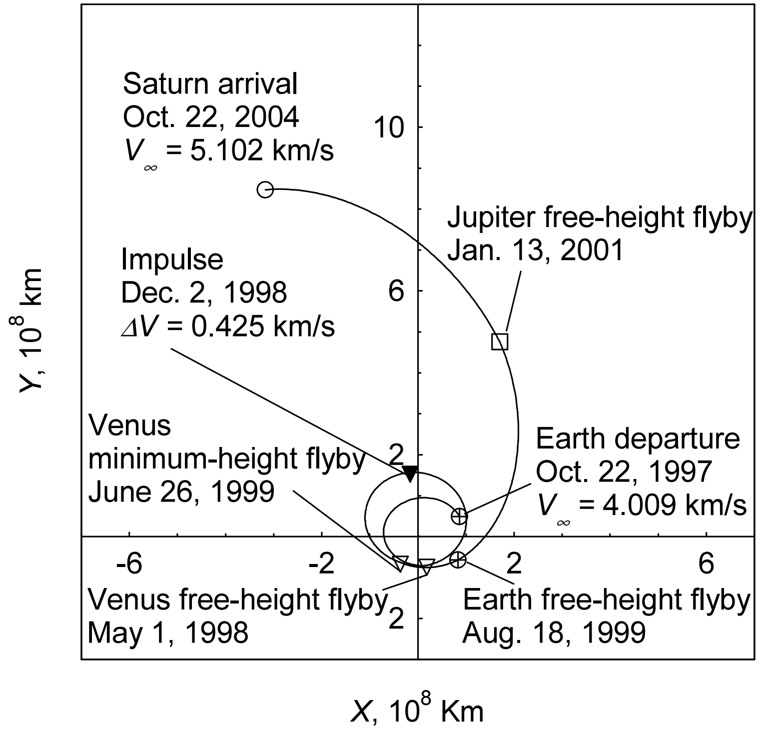
Comparison of Basic and Hybrid Algorithms (3)

- hybrid algorithms with NON optimal settings

algorithm	ϵ	N_{min}	N_{mean}	S_N
DE	0.80	20884	67122	36268
GA	0.58	71021	227527	1870
PSO	0.30	9022	196702	25690
GA + DE	0.98	101426	146743	24578
GA + PSO	0.88	62429	427379	14353
DE + PSO	0.90	26938	57064	24922
GA + DE + PSO	1.00	97048	177441	18595

Cassini Mission (1)

- EVVEJS mission: departure from Earth, flybys of Venus (twice), Earth and Jupiter, arrival at Saturn
- DS impulse during VV leg
- $\Delta V = 4.892$ km/s



Cassini Mission (2)

- one impulse allowed (defined by position), 10 parameters
- DE with optimal settings from Olds and Kluever

algorithm	N	ϵ	t, s	$t_{95\%}$, s	N_{avg}
DE	112000	0.16	10.5	194	35637
DE MM	112000	0.15	10.8	201	41065
Hybrid	300000	0.16	25.0	462	153532
DE	300000	0.20	29.0	389	55099
DE MM	300000	0.37	30.1	195	96470

Cassini Mission (3)

- one impulse allowed (defined by leg initial velocity, reference is made to departure velocity of the ballistic arc that joins the planets)

algorithm	N	ϵ	t, s	$t_{95\%}$, s	N_{avg}
DE	112000	0.39	19.5	118	17669
DE MM	112000	0.89	16.8	23	38513
Hybrid	300000	0.82	40.8	71	77334
DE	300000	0.40	51.5	302	17510
DE MM	300000	1.00	45.3	45	48222

Cassini Mission (4)

- three impulses allowed, 18 parameters
- convergence obtained only when impulses are defined by leg initial velocity

algorithm	F	ϵ	t, s	$t_{95\%}$, s	N_{avg}
DE	0.6	0.20	104.2	1398	84127
DE MM	0.6	0.36	94.6	635	157803
Hybrid	0.6	0.41	66.2	376	307593
DE	[-1,1]	0.35	101.5	706	47514
DE MM	[-1,1]	0.67	92.2	249	97206
Hybrid	[-1,1]	0.64	84.1	247	255136

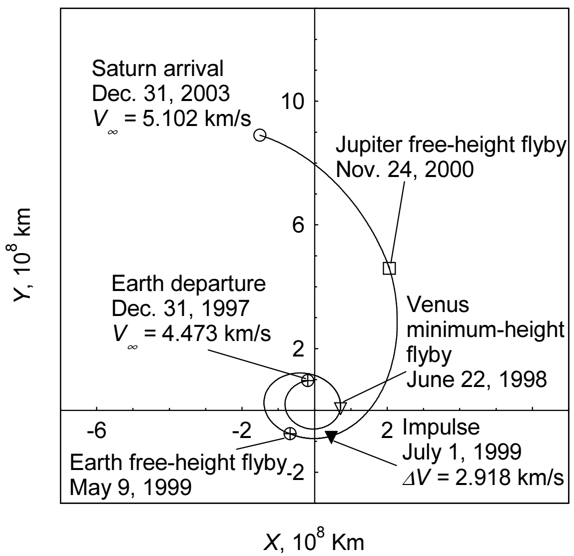
Cassini Mission (5)

- five impulses allowed, 26 parameters
- convergence obtained only when impulses are defined by leg initial velocity

algorithm	F	ϵ	t, s	$t_{95\%}$, s	N_{avg}
DE MM	0.6	0.35	207.1	1440	485626
Hybrid	0.6	0.41	199.7	1134	411157
DE MM	[-1, 1]	0.51	220.1	924	156375
Hybrid	[-1, 1]	0.53	181.4	720	392854

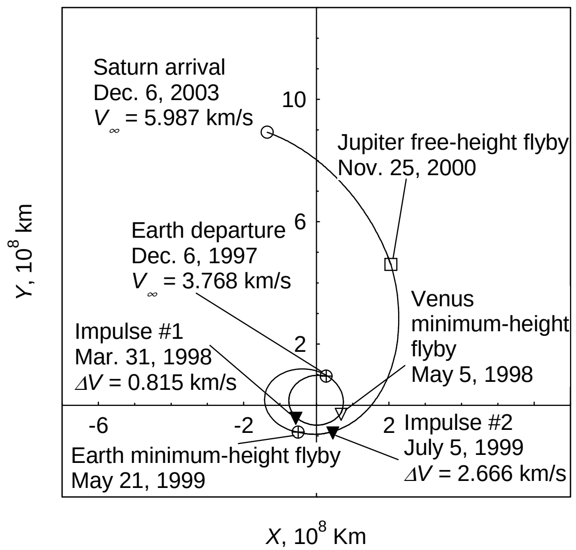
Fast Saturn Mission (1)

- EVEJS mission: departure from Earth, flybys of Venus, Earth and Jupiter, arrival at Saturn



optimal (1 imp.)

$\Delta V = 7.682$ km/s



suboptimal (2 imps.)

$\Delta V = 8.024$ km/s

Fast Saturn Mission (2)

- up to four impulses allowed, 21 parameters
- convergence obtained only when impulses are defined by leg initial velocity

algorithm	F	ΔV_m, km/s	ϵ	t, s	$t_{95\%}$, s	N_{avg}
DE	0.6	7.682	0.01	71.5	21312	44703
DE MM	0.6	7.705	0.03	78.1	7681	44389
Hybrid	0.6	7.682	0.09	81.9	2602	224263
DE	[-1, 1]	7.682	0.03	73.1	7190	20051
DE MM	[-1, 1]	7.682	0.03	78.5	7720	20067
Hybrid	[-1, 1]	7.682	0.05	81.5	4760	230802

Conclusions

- importance of problem formulation (choice of design variables)
- improvements provided by mass mutation (and extended random initialization)
- cooperative approach allows good performance with "average" settings for the algorithm parameters
- no specific parameter tuning is required
- immediate application to new problems
- benefit more remarkable as problem complexity grows and (quite obviously) when nonoptimal parameter settings are adopted
- current and future work: search for opportunities for future missions / optimize planet sequence

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Thanks for your attention

Questions are welcome