Modeling orbital relative motion to enable formation design from application requirements

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Introduction

- Formation flying: achieving the functionality of a very large satellite with multiple small satellites. Cost: deep complication of system dynamics

- A significant effort has been made in the latest years to improve knowledge of formation dynamics, mainly with the goal of formation control

- Inclusion of J_2 perturbation in relative motion analysis and control has been performed with different approaches by many authors, for example *Schweighart and Sedwick, Wiesel, Vadali et al*., *Alfriend et al., Schaub*

- From mission design point of view, there is a substancial lack in literature of time-explicit design-oriented models

- The gap between application requirements and orbital dynamics can be bridged by means of the description of relative motion in terms of differences in mean orbital parameters

Problem formulation and assumptions

- 2 satellites, *chief* and *deputy*

- Goal: to describe relative motion (deputy in chief Hill reference frame) both in close and in large formations (inter-satellite separation of hundreds of kilometers) → quadratic model (linear as a particular case)

- Design-framework: stable or slowly drifting formations in low eccentricity low Earth orbits → Inclusion of J_2 secular effects is performed

  $\Delta a/a$, Δe and Δi are constant in time, while $\Delta\Omega$, ΔM and $\Delta\omega$ can be larger without impacting formation stability and can change if differential effects exist: different orders of approximation are needed to take realistic cases into account

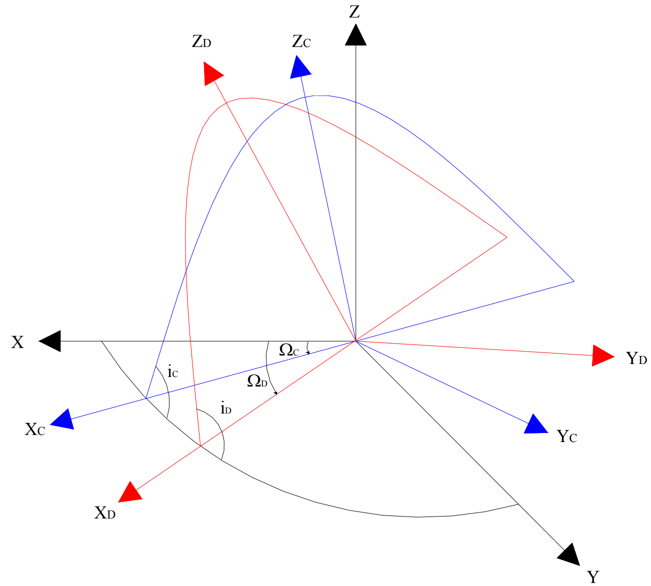
- In general, it can be supposed that $\Delta a/a$ is designed as the smallest parameter (formation instability even in keplerian dynamics)

Problem formulation and assumptions

Exact equations of relative motion in the orbital parameters approach:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M_{CH} \cdot M_{DC} \cdot r_D \cdot \begin{bmatrix} \cos u_D \\ 0 \\ \sin u_D \end{bmatrix} - \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix}$$

$$r_D = \frac{a_D\left(1 - e_D^{\ 2}\right)}{1 + e_D \cos \nu_D} \qquad r = \frac{a\left(1 - e^2\right)}{1 + e \cos \nu}$$



$$M_{DC} = \begin{bmatrix} \cos \Delta\Omega & -\sin i_D \sin \Delta\Omega & -\cos i_D \sin \Delta\Omega \\ \sin i \sin \Delta\Omega & \begin{matrix}\sin i \sin i_D \cos \Delta\Omega + \\ + \cos i \cos i_D\end{matrix} & \begin{matrix}\sin i \cos i_D \cos \Delta\Omega + \\ - \cos i \sin i_D\end{matrix} \\ \cos i \sin \Delta\Omega & \begin{matrix}\cos i \sin i_D \cos \Delta\Omega + \\ - \sin i \cos i_D\end{matrix} & \begin{matrix}\cos i \cos i_D \cos \Delta\Omega + \\ + \sin i \sin i_D\end{matrix} \end{bmatrix}$$

$$M_{CH} = \begin{bmatrix} \cos(\omega + \nu) & 0 & \sin(\omega + \nu) \\ -\sin(\omega + \nu) & 0 & \cos(\omega + \nu) \\ 0 & -1 & 0 \end{bmatrix}$$

Relative motion for chief on circular orbit

- On the basis of previous considerations :
- Second order approximation for $\Delta\Omega$, $\omega_{d0}+M_{d0}-u_0$, $\Delta \dot{u}\, t = \Delta\left(\dot{M}+\dot{\omega}\right)t$
- First order approximation for Δi.

Exact equations

Small $\Delta\Omega$ and Δi

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \cong \begin{bmatrix} \left(1-\frac{\Delta\Omega^2}{2}+\frac{\Delta\Omega^2}{4}\sin^2 i\right)\cos\left(u_D-u\right)+\left(\frac{1}{2}\Delta\Omega\Delta i\sin i-\Delta\Omega\cos i\right)\sin\left(u_D-u\right)-\frac{\Delta\Omega^2}{4}\sin^2 i\cos\left(u_D+u\right)+\frac{1}{2}\Delta\Omega\Delta i\sin i\sin\left(u_D+u\right) \\ \left(1-\frac{\Delta\Omega^2}{2}+\frac{\Delta\Omega^2}{4}\sin\ i\right)\sin\left(u_D-u\right)+\left(\Delta\Omega\cos i-\frac{1}{2}\Delta\Omega\Delta i\sin i\right)\cos\left(u_D-u\right)+\frac{\Delta\Omega^2}{4}\sin\ i\sin\left(u_D+u\right)+\frac{1}{2}\Delta\Omega\Delta i\sin i\cos\left(u_D+u\right) \\ -\Delta\Omega\sin i\cos u_D+\left(\Delta i+\frac{\Delta\Omega^2}{2}\sin i\cos i\right)\sin u_D \end{bmatrix} r_D-\begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$$

Then, we have to expand in power series of eccentricity and arrest at first order, apply the assumption of small along-track separation, and consider that mean anomaly, argument of perigee and right ascension of the ascending node must be properly expressed as initial value + secular variation caused by J_2

Relative motion for chief on circular orbit

For example, we get

$$r_D \cos(u_D - u) = \frac{a_D\left(1 - e_D^{\,2}\right)}{1 + e_D \cos \nu_D} \cos\left(\omega_D + \nu_D - \dot{u}\,t - u_0\right) \cong a_D \cdot \left\{1 - \frac{(\Delta u_0)^2}{2} - \frac{\left(\Delta \dot{u}\,t\right)^2}{2} - \Delta u_o \Delta \dot{u}\,t - e_D \cdot \right.$$

$$\left. [\cos\left(M_{D0} + \dot{M}_D\,t\right) + 2 \sin\left(M_{D0} + \dot{M}_D\,t\right)\left(\Delta \dot{u}\,t + \Delta u_0\right)] \right\}$$

$$u_0 = \omega_0 + M_0$$

$$\Delta u_0 = \Delta(\omega_0 + M_0)$$

Similar expansions can be generated for all the other products $r_D\sin(u_D+u)$, $r_D\cos(u_D+u)$, $r_D\sin u_D$, and $r_D\cos u_D$.
By introducing these expansions in previous relations, taking into account that

$$a_D = a + \Delta a$$

and neglecting terms of higher order than the second, we can derive the 2nd order relative motion equations in final form

Relative motion for chief on circular orbit

$$\frac{x}{a} \cong \frac{\Delta a}{a} - e_d \cos\left(M_{d0} + \dot{M}_d t\right) - \frac{(\Delta u_0)^2}{2} - \frac{\left(\Delta \dot{u} t\right)^2}{2} - \Delta u_0 \cdot \Delta \dot{u} t - e_d\left(\Delta u_0 + \Delta \dot{u} t\right) 2 \sin\left(M_{d0} + \dot{M}_d t\right) +$$
$$- \Delta\Omega \cos i [\Delta u_0 + \Delta \dot{u} t + 2 e_d \sin\left(M_{d0} + \dot{M}_d t\right)] + \left(-\frac{\Delta\Omega^2}{2} + \frac{\Delta\Omega^2}{4} \sin^2 i\right) - \frac{\Delta\Omega^2}{4} \sin^2 i \cos\xi + \frac{1}{2}\Delta\Omega\Delta i \sin i \sin\xi$$

$$\frac{y}{a} \cong 2 e_d \sin\left(M_{d0} + \dot{M}_d t\right) + \Delta u_0 + \Delta \dot{u} t + \Delta\Omega \cos i - e_d\left(\Delta u_0 + \Delta \dot{u} t\right)\cos\left(M_{d0} + \dot{M}_d t\right) - \Delta\Omega \cos i e_d$$
$$\cos\left(M_{d0} + \dot{M}_d t\right) - \frac{1}{2}\Delta\Omega\Delta i \sin i + \frac{\Delta\Omega^2}{4} \sin^2 i \cdot \sin\xi + \frac{1}{2}\Delta\Omega\Delta i \sin i \cos\xi$$

$$\frac{z}{a} \cong -\Delta\Omega \sin i \cos\psi + \Delta i \sin\psi - \Delta\Omega e_d \sin i [\frac{1}{2}\cos(2M_{d0} + \omega_{d0} + \dot{M}_d t + \dot{u}_d t) - \frac{3}{2}\cos\left(\omega_{d0} + \dot{\omega}_d t\right)] + \Delta i e_d \cdot$$
$$\cdot \left[\frac{1}{2}\sin\left(2M_{d0} + \omega_{d0} + \dot{M}_d t + \dot{u}_d t\right) - \frac{3}{2}\sin\left(\omega_{d0} + \dot{\omega}_d t\right)\right] + \frac{\Delta\Omega^2}{2} \sin i \cos i \sin\psi$$

$$\xi = \omega_{d0} + M_{d0} + u_0 + 2\dot{u}_d t$$

$$\psi = \omega_{d0} + M_{d0} + \dot{u}_d t$$

Relative motion for chief on circular orbit

First order terms (close formations)

- Neglecting J_2 equations are consistent with Hill's solution, showing for example that in-plane trajectory is an ellipse of fixed eccentricity. Including J_2:
- Δa still appears as a **radial offset**, whereas it contributes to secular terms in y and z, together with Δe and Δi
- **In-plane coordinates and z exhibit different frequencies**. In fact, the radial/along-track motion is phased relative to the perigee, while the cross-track motion is phased relative to the nodal crossings. This originates the "tumbling" phenomenon (even for J_2-invariant orbits)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \cong a \begin{bmatrix} \dfrac{\Delta a}{a} - e_D \cos(M_{D0} + \dot{M}_D t) \\ 2e_d \sin(M_{D0} + \dot{M}_D t) + (\omega_{D0} + M_{D0} - u_0) + \\ + \Delta\Omega_0 \cos i + (\Delta \dot{u} + \Delta \dot{\Omega} \cos i) t \\ -(\Delta\Omega_0 + \Delta\dot{\Omega} t) \sin i \cos(\omega_{D0} + M_{D0} + \dot{u}_D t) + \\ + \Delta i \sin(\omega_{D0} + M_{D0} + \dot{u}_D t) \end{bmatrix}$$

Keplerian case: consistent with Hill's solution

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \cong a \begin{bmatrix} \dfrac{\Delta a}{a} - e_D \cos(M_{D0} + n_D t) \\ 2e_D \sin(M_{D0} + n_D t) + (\omega_D + M_{D0} - u_0) + \\ + \Delta\Omega \cos i + (\Delta n) t \\ -\Delta\Omega \sin i \cos(\omega_D + M_{D0} + n_D t) + \\ + \Delta i \sin(\omega_D + M_{D0} + n_D t) \end{bmatrix}$$

Relative motion for chief on circular orbit

Second order terms have a coupling effect on the coordinates. They are:

Offsets (only in-plane components)

Drifts: linear or quadratic function of time (only in-plane components)

Oscillations (constant or slowly time-varying amplitude) with period that equals anomalistic (x and y) or nodal (z) period

Oscillations (constant or slowly time-varying amplitude) at frequency twice as the orbital frequency

Long period oscillations (constant or slowly time-varying amplitude) due to perigee precession

$$\frac{x}{a} \cong \frac{\Delta a}{a} - e_d \cos\left(M_{d0} + \dot{M}_d t\right) - \frac{(\Delta u_0)^2}{2} - \frac{\left(\Delta \dot{u} t\right)^2}{2} - \Delta u_0 \cdot \Delta \dot{u} t - e_d \left(\Delta u_0 + \Delta \dot{u} t\right) 2 \sin\left(M_{d0} + \dot{M}_d t\right) +$$
$$- \Delta\Omega \cos i [\Delta u_0 + \Delta \dot{u} t + 2 e_d \sin\left(M_{d0} + \dot{M}_d t\right)] + \left(-\frac{\Delta\Omega^2}{2} + \frac{\Delta\Omega^2}{4} \sin^2 i\right) - \frac{\Delta\Omega^2}{4} \sin^2 i \cos \xi + \frac{1}{2} \Delta\Omega\Delta i \sin i \sin \xi$$

$$\frac{y}{a} \cong 2 e_d \sin\left(M_{d0} + \dot{M}_d t\right) + \Delta u_0 + \Delta \dot{u} t + \Delta\Omega \cos i - e_d \left(\Delta u_0 + \Delta \dot{u} t\right) \cos\left(M_{d0} + \dot{M}_d t\right) - \Delta\Omega \cos i e_d$$
$$\cos\left(M_{d0} + \dot{M}_d t\right) - \frac{1}{2} \Delta\Omega\Delta i \sin i + \frac{\Delta\Omega^2}{4} \sin^2 i \cdot \sin \xi + \frac{1}{2} \Delta\Omega\Delta i \sin i \cos \xi$$

$$\frac{z}{a} \cong -\Delta\Omega \sin i \cos \psi + \Delta i \sin \psi - \Delta\Omega e_d \sin i [\frac{1}{2} \cos(2 M_{d0} + \omega_{d0} + \dot{M}_d t + \dot{u}_d t) - \frac{3}{2} \cos\left(\omega_{d0} + \dot{\omega}_d t\right)] + \Delta i e_d \cdot$$
$$\cdot \left[\frac{1}{2} \sin\left(2 M_{d0} + \omega_{d0} + \dot{M}_d t + \dot{u}_d t\right) - \frac{3}{2} \sin\left(\omega_{d0} + \dot{\omega}_d t\right)\right] + \frac{\Delta\Omega^2}{2} \sin i \cos i \sin \psi$$

Relative motion for chief on low eccentricity orbit

Approach: consider a reference virtual platform, which moves on a circular orbit sharing all chief orbital parameters except eccentricity (and of course, in time, true anomaly).

Reference frames:

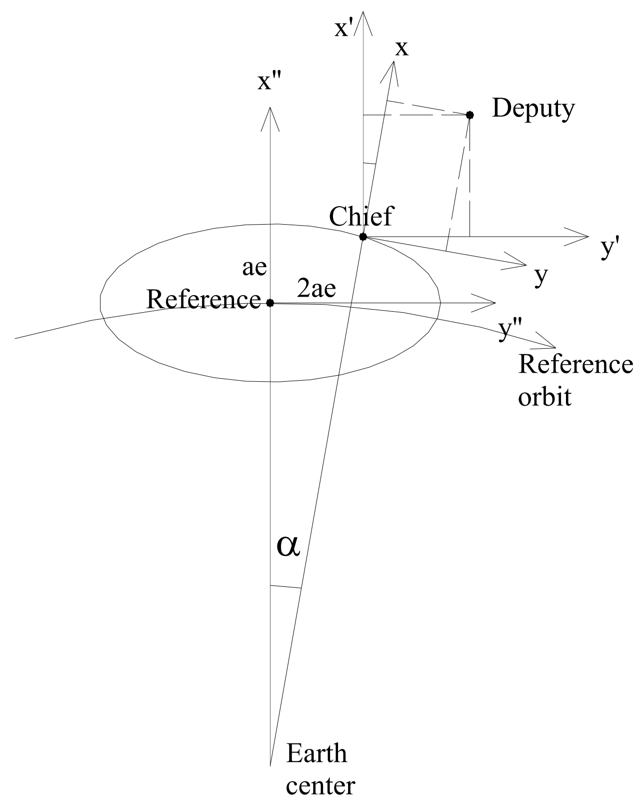
$x^{II}y^{II}$ is reference satellite HRF,

xy is chief HRF,

$x^{I}y^{I}$ is the reference frame which is obtained from $x^{II}y^{II}$ by means of a translation in the chief position. xy is obtained from $x^{I}y^{I}$ by means of a rotation α.

Chief motion w.r.t. reference:

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} \cong a \begin{bmatrix} -e\cos(M_0 + \dot{M}t) \\ 2e\sin(M_0 + \dot{M}t) + (\Delta\dot{u} + \Delta\dot{\Omega}\cos i)t \\ -\Delta\dot{\Omega}t\sin i\cos(\omega_0 + M_0 + \dot{u}t) \end{bmatrix}$$



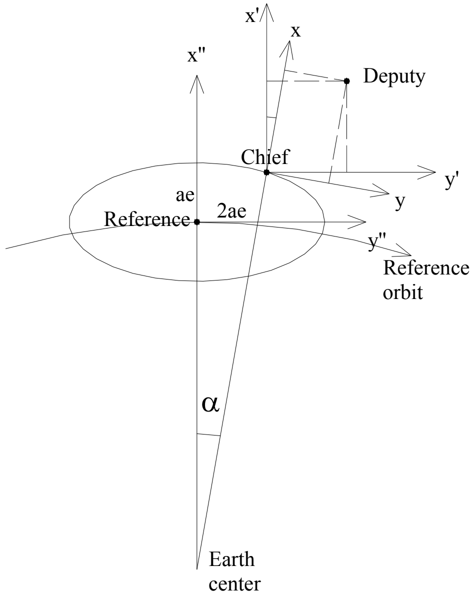
Relative motion for chief on low eccentricity orbit

Deputy motion w.r.t. reference: 2nd order equations

Relative motion:

Make the difference between deputy and chief components → $\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$

Apply rotation α → $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$



In general

$$x = x' \cos\alpha + y' \sin\alpha$$
$$y = -x' \sin\alpha + y' \cos\alpha$$

$$\alpha = arctg \frac{y_{chief-reference}}{a + x_{chief-reference}}$$

In our first order approximation in eccentricity

$$\alpha \cong \frac{y_{chief-reference}}{a}$$

$$\cos\alpha \cong 1$$
$$\sin\alpha \cong \alpha$$

Second order terms in eccentricity are neglected, but

<u>No assumptions are considered on the difference in mean anomaly and argument of perigee</u>: the sum of the two differences can be approximated at second order

Relative motion for chief on low eccentricity orbit

First order terms related to chief eccentricity: in-plane oscillation due to the difference in mean anomaly between deputy and chief. Of higher order for small mean anomaly differences.

Second order terms related to chief eccentricity

$$\frac{x^{|}}{a} \cong \frac{\delta a}{a} - \delta e \cos\left(M_{D0} + \dot{M}_D t\right) + \underline{2e\sin\left(\frac{\delta M_0}{2}\right)\sin\left(M_0 + \frac{\delta M_0}{2} + \dot{M}_D t\right)} + \underline{e\left(\delta \dot{M} t\right)\sin\left(M_0 + \dot{M}_D t\right)} +$$

$$- \frac{(\delta u_0)^2}{2} - \frac{\left(\delta \dot{u} t\right)^2}{2} - \delta u_0 \cdot \delta \dot{u} t - e_d\left(\delta u_0 + \delta \dot{u} t\right) 2\sin\left(M_{d0} + \dot{M}_d t\right) - \delta\Omega \cos i[\delta[_0 + \delta \dot{u} t + 2e_d \sin\left(M_{d0} + \dot{M}_d t\right)]$$

$$+ \left(-\frac{\delta\Omega^2}{2} + \frac{\delta\Omega^2}{4}\sin^2 i\right) - \frac{\delta\Omega^2}{4}\sin^2 i \cos\xi + \frac{1}{2}\delta\Omega\,\delta i \sin i \sin\xi + \underline{2e\sin\left(M_0 + \dot{M} t\right)\left(\delta u_0 + \delta \dot{u} t + \delta\Omega \cos i\right)}$$

$$\frac{y^{|}}{a} \cong \delta u_0 + \delta \dot{u} t + \delta\Omega \cos i + 2\delta e \sin\left(M_{D0} + \dot{M}_D t\right) + \underline{4e\sin\left(\frac{\delta M_0}{2}\right)\cos\left(M_0 + \frac{\delta M_0}{2} + \dot{M}_D t\right)} +$$

$$\underline{+ 2e\left(\delta \dot{M} t\right)\cos\left(M_0 + \dot{M}_D t\right)} - e_d\left(\delta u_0 + \delta \dot{u} t\right)\cos\left(M_{d0} + \dot{M}_d t\right) - \delta\Omega \cos i e_d \cos\left(M_{d0} + \dot{M}_d t\right) +$$

$$- \frac{1}{2}\delta\Omega\,\delta i \sin i + \frac{\delta\Omega^2}{4}\sin^2 i \cdot \sin\xi + \frac{1}{2}\delta\Omega\,\delta i \sin i \cos\xi$$

$$\frac{z^{|}}{a} \cong -\delta\Omega \sin i \cos\psi + \delta i \sin\psi - \delta\Omega\, e_d \sin i[\frac{1}{2}\cos(2M_{d0} + \omega_{d0} + \dot{M}_d t + \dot{u}_d t) - \frac{3}{2}\cos\left(\omega_{d0} + \dot{\omega}_d t\right)] +$$

$$+ \delta i e_d\left[\frac{1}{2}\sin\left(2M_{d0} + \omega_{d0} + \dot{M}_d t + \dot{u}_d t\right) - \frac{3}{2}\sin\left(\omega_{d0} + \dot{\omega}_d t\right)\right] + \frac{\delta\Omega^2}{2}\sin i \cos i \sin\psi$$

Relative motion for chief on low eccentricity orbit

- Useful development for mission design: expressing secular terms as linear functions of Δa, Δe, and Δi

$$\Delta\dot{\Omega} = \Delta\dot{\Omega}(a, e, i, \Delta a, \Delta e, \Delta i)$$

$$\Delta\dot{u} = \Delta\dot{u}(a, e, i, \Delta a, \Delta e, \Delta i)$$

$$\Delta\dot{\Omega} = \frac{\partial\dot{\Omega}}{\partial a}\bigg|_{a,e,i}\Delta a + \frac{\partial\dot{\Omega}}{\partial e}\bigg|_{a,e,i}\Delta e + \frac{\partial\dot{\Omega}}{\partial i}\bigg|_{a,e,i}\Delta i$$

$$\Delta\dot{u} = \frac{\partial\dot{u}}{\partial a}\bigg|_{a,e,i}\Delta a + \frac{\partial\dot{u}}{\partial e}\bigg|_{a,e,i}\Delta e + \frac{\partial\dot{u}}{\partial i}\bigg|_{a,e,i}\Delta i$$

$$\Delta\dot{\Omega} = \dot{\Omega}\left(C_a\frac{\Delta a}{a} + C_e\Delta e + C_i\Delta i\right)$$

$$\Delta\dot{u} = \dot{\Omega}\left(\xi_a\frac{\Delta a}{a} + \xi_e\Delta e + \xi_i\Delta i\right)$$

$$C_e = \frac{4e}{1-e^2}$$

$$\xi_e = \left(\frac{4\dot{\omega} + 3\dot{M}_p}{\dot{\Omega}}\right)\frac{e}{1-e^2}$$

$$C_a = -\frac{7}{2}$$

$$C_i = -\tan i$$

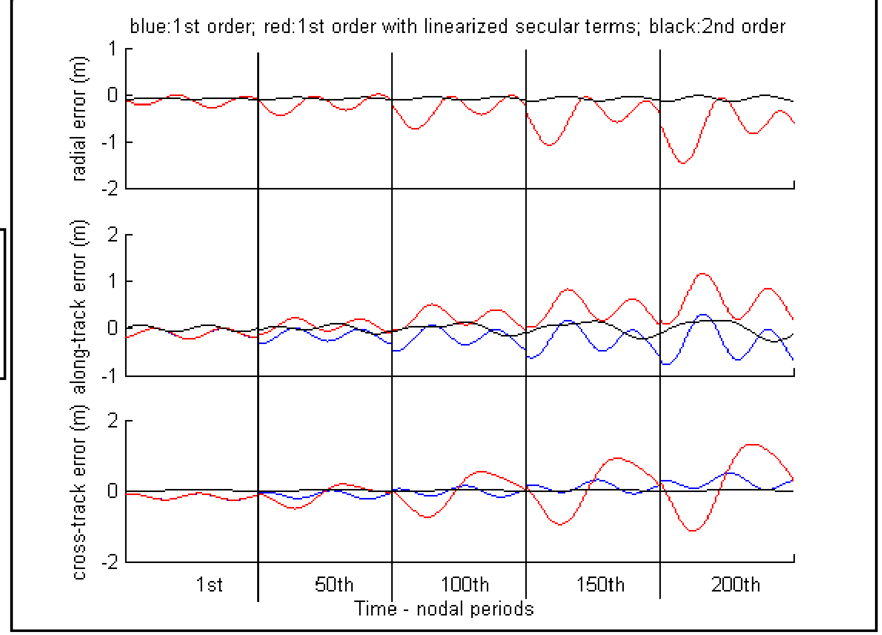
$$\xi_a = -\left(\frac{3\dot{u}}{2\dot{\Omega}} + 2\frac{\dot{M}_p + \dot{\omega}}{\dot{\Omega}}\right)$$

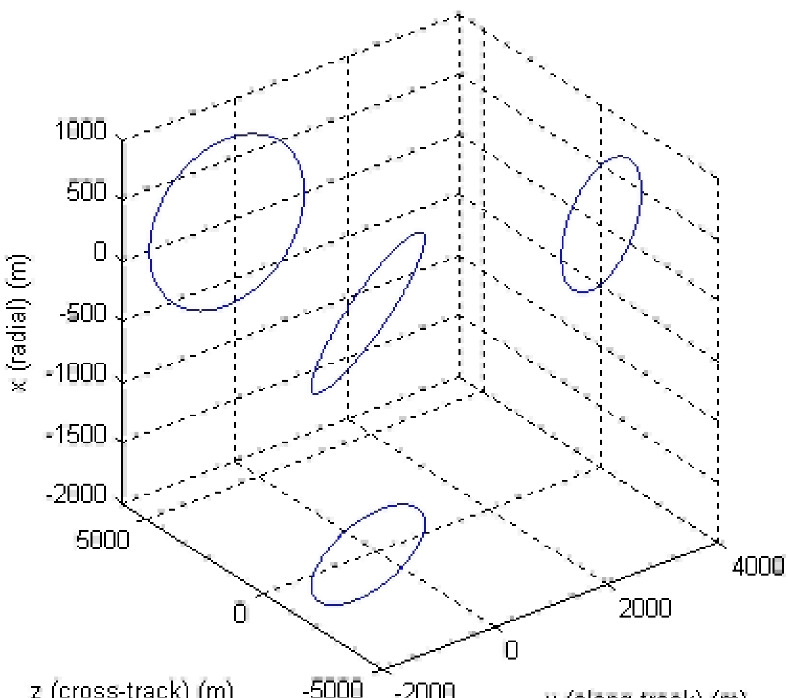
$$\xi_i = -\left(\frac{3}{1-\frac{3}{2}\sin^2 i}\cdot\frac{\dot{M}_p}{\dot{\Omega}} + \frac{5}{2-\frac{5}{2}\sin^2 i}\cdot\frac{\dot{\omega}}{\dot{\Omega}}\right)\sin i\cos i$$

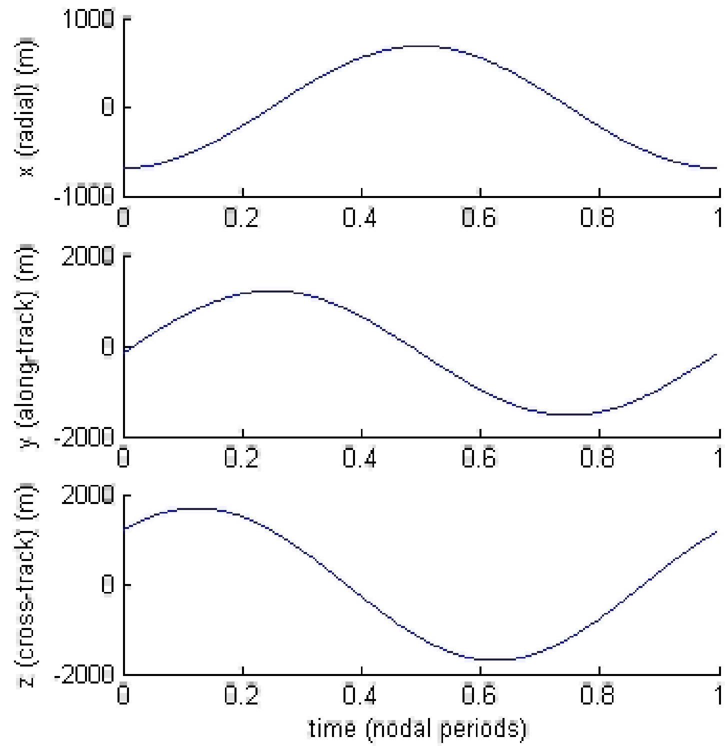
Of course, the approximation is better for close formations

Coefficients show for example the small dependence of J_2 effects on eccentricity for near circular orbits

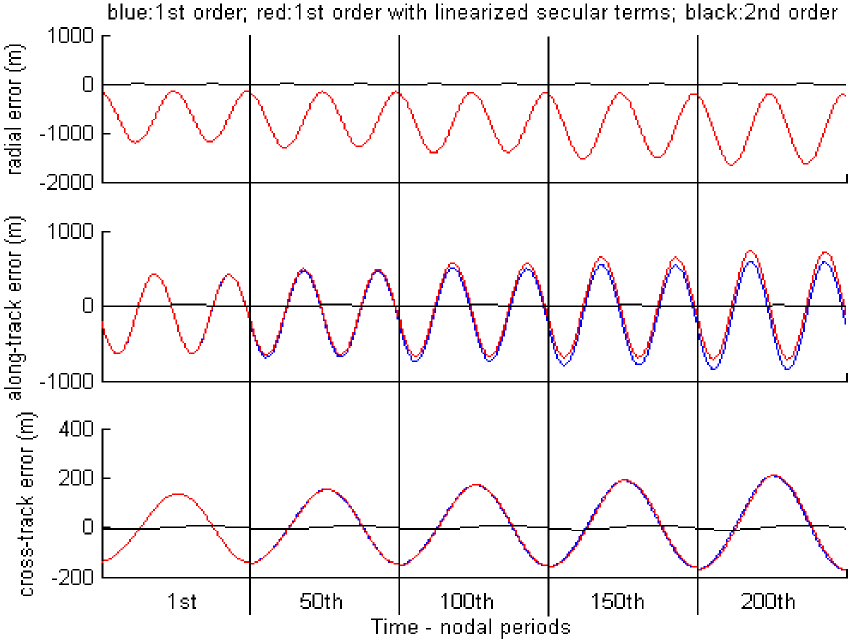
Modeling error

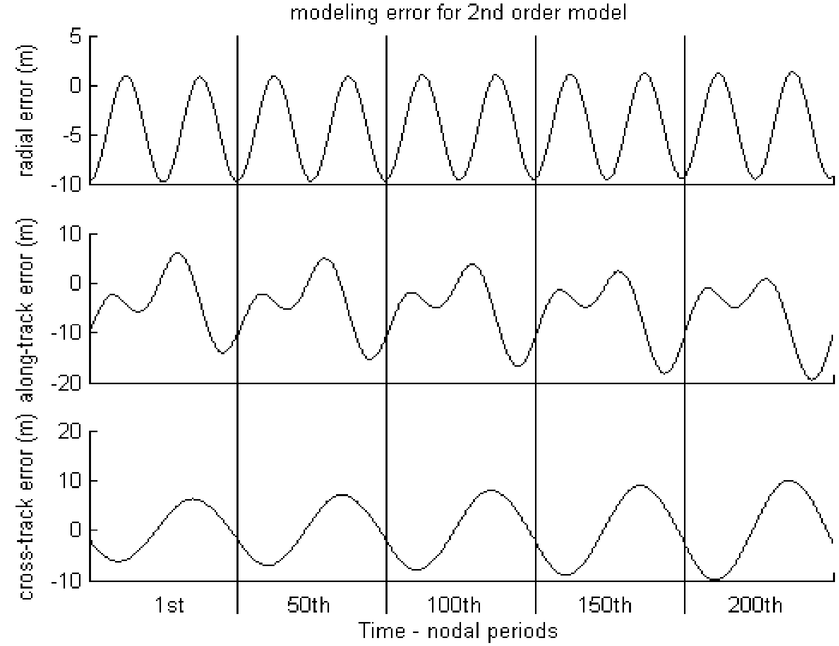


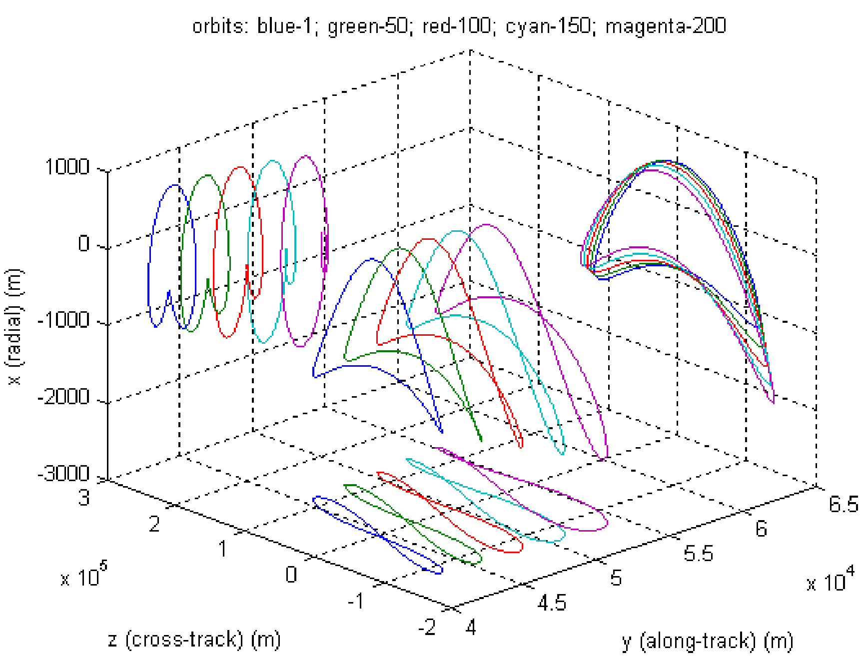


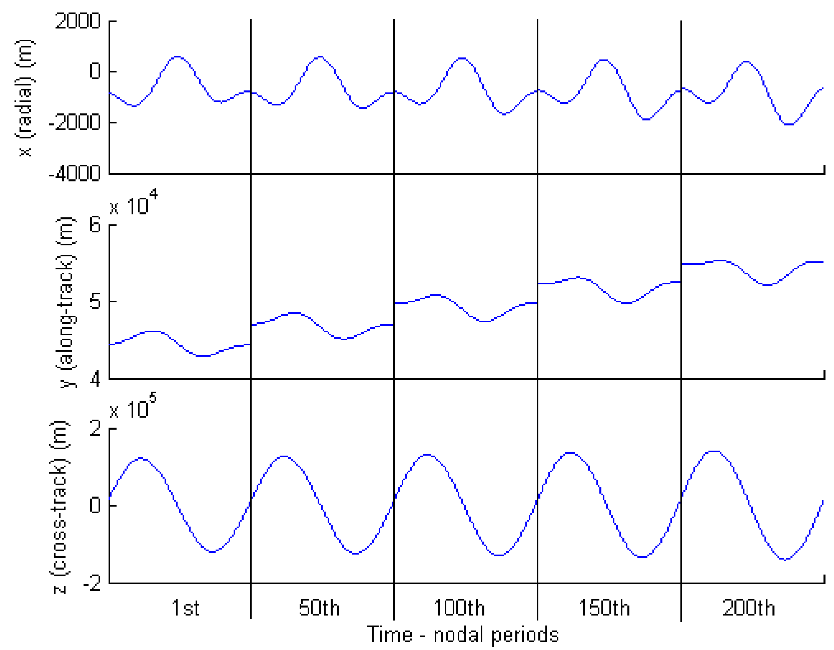


Modeling error







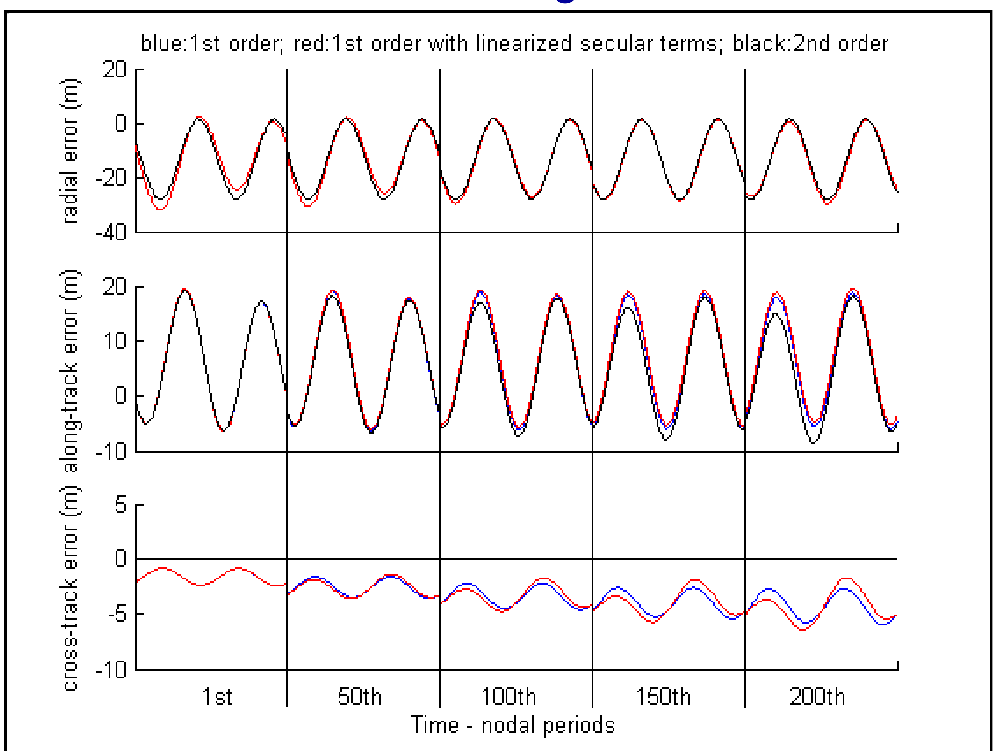


Evaluation of modeling accuracy

Third test case: both satellites on low eccentricity orbits, close formation (with large difference in mean anomaly)

Initial orbital parameters	Chief	Deputy
Semi-major axis (km)	7000	7000
Eccentricity	0.001	0.0009
Inclination (°)	97.87	97.88
Right Asc. of the A.N.(°)	0	0.01
Perigee anomaly (°)	90	180
True anomaly (°)	0	269.886

Modeling error: in the order of 0.1% of relative motion coordinates on long timescales

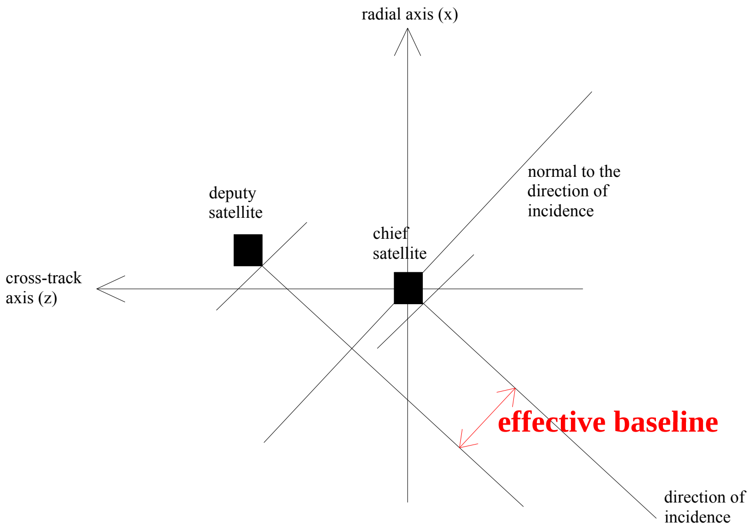


Formation design case study

➢ Two satellite mission devoted to cross-track interferometric SAR (chief on low eccentricity orbit)

➢ The basic formation parameter which determines observation geometry and achievable performance in DEM generation is the so-called "effective baseline", which is the projection of baseline onto the normal of radar viewing direction. Usually, an optimal value/range can be identified

➢ Orbital motion produces a continuous change of effective baseline: formation design can be based on a range of latitudes where a given performance is requested. However, if differences in orbital parameters are properly chosen, a quite stable effective baseline can be achieved at almost all the latitudes along the orbit.



$$\left| x \sin \vartheta - z \cos \vartheta \right|$$

(θ is radar look angle)

From formation design point of view, we have to exploit and properly combine horizontal and vertical baseline.

Formation design case study

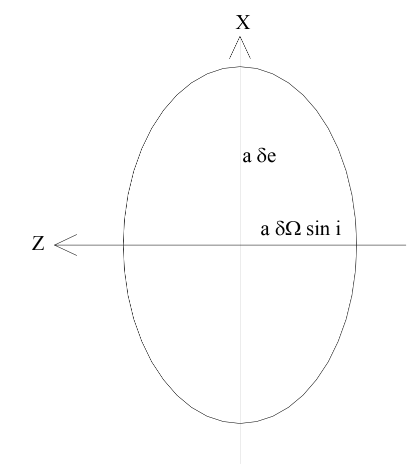
No choice on the horizontal component: imposing the required δi would result in cross-track instability. Thus, **horizontal motion has to be obtained by means of δΩ**

Two choices for the vertical oscillation: using **differences in mean anomaly and in argument of perigee** and/or **a difference in eccentricity**

In the second case the phase difference between vertical and horizontal oscillation is equal to ω_D

$$\begin{bmatrix} x\\ y\\ z\end{bmatrix} \cong a \begin{bmatrix} \frac{\delta a}{a} - \delta e\cos(M_0+\delta M_0+\dot{M}_D t) + \\ + 2e\sin\left(\frac{\delta M_0}{2}\right)\sin\left(M_0+\dot{M}_D t+\frac{\delta M_0}{2}\right) \\ 2\delta e\sin(M_0+\delta M_0+\dot M_D t) + \\ + 4e\sin\frac{\delta M_0}{2}\cos\left(M_0+\frac{\delta M_0}{2}+\dot M_D t\right) + \\ + \delta(\omega_0+M_0)+\delta\Omega_0\cos i + t\left(\delta\dot u+\delta\dot\Omega\cos i\right) \\ -(\delta\Omega_0+\delta\dot\Omega t)\sin i\cos(\omega_{D0}+M_{D0}+\dot u_D t) + \\ + \delta i\sin(\omega_{D0}+M_{D0}+\dot u_D t)\end{bmatrix}$$

In a cross-track INSAR mission, argument of perigee is likely to be 90°: if this is the case, imposing a δe implies that xz trajectory is an ellipse whose principal directions coincide with the coordinate axes and whose semi-major axes depend linearly on δΩ and δe

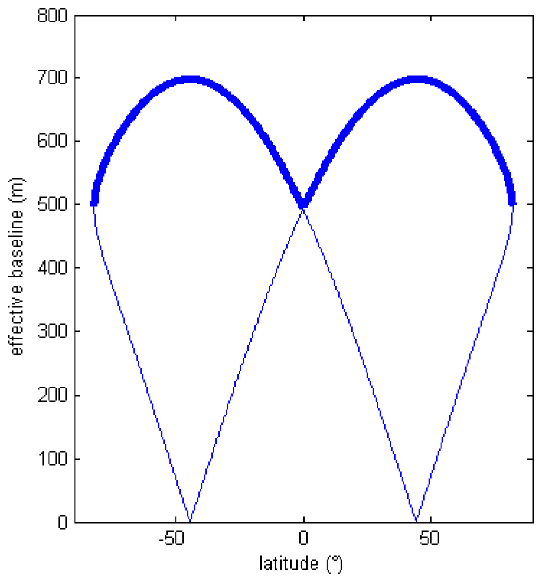


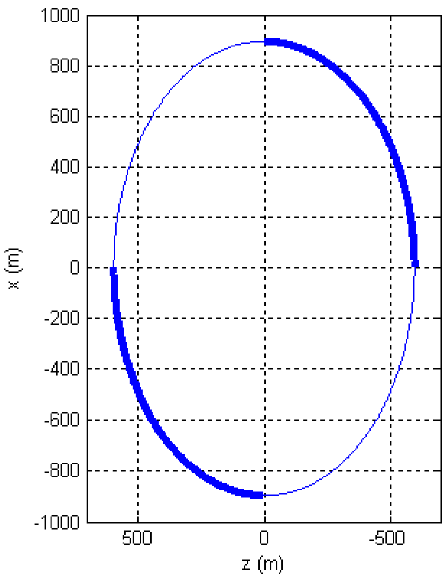
Formation design case study

The formation allows for stable effective baselines at all the achievable latitudes along the orbit. The ellipse can be shaped with simple calculations on the basis of the requested effective baseline range and the latitude where a given value is desired.

For example, it is possible to maximize the orbit fraction where the required effective baseline range is achieved, and to have the maximum effective baseline at intermediate latitudes.

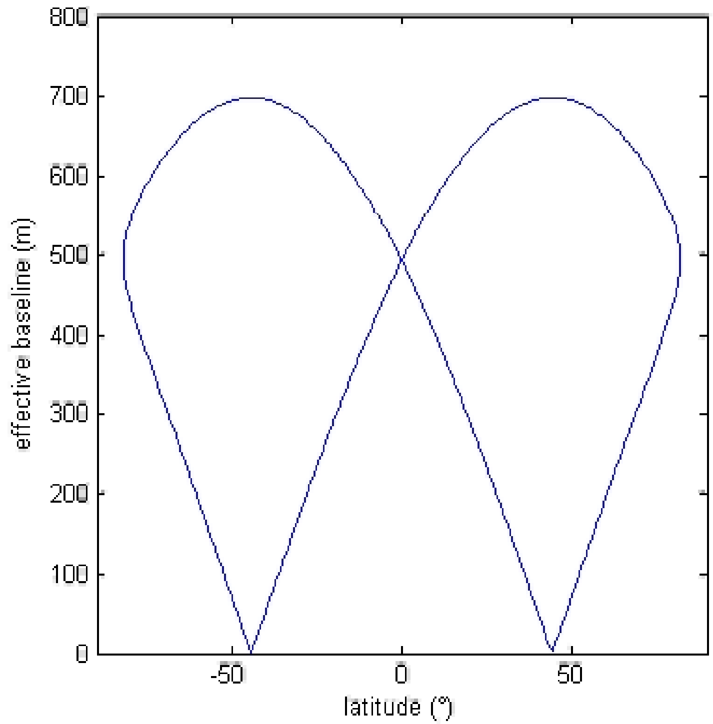
Assuming for example COSMO/Skymed as chief satellite, an effective baseline range of 400 m ÷ 700 m, and radar off-nadir angle equal to 33.5°, it is possible to derive $\delta\Omega = \pm 0.0049°$, $\delta e = \pm 1.28 \cdot 10^{-4}$.

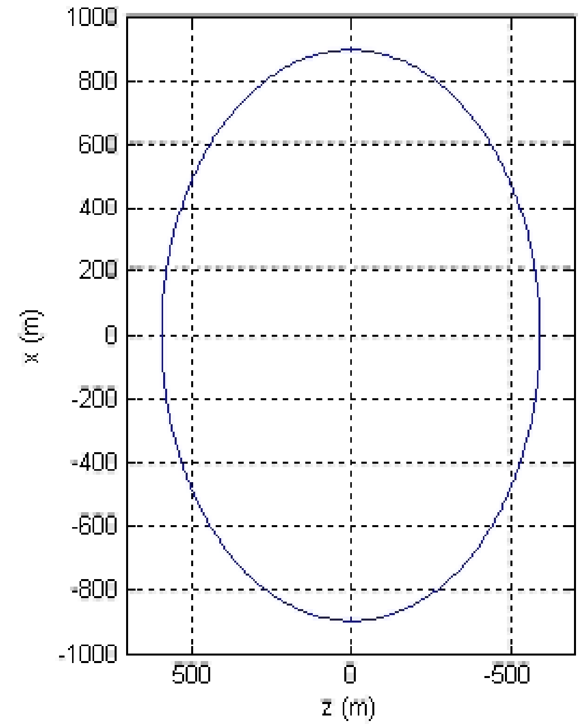




Formation design case study

The considered formation is not J_2 invariant, but differential J_2 effects on the single coordinates are very small (δe). Line of apsides precession poses instead a major problem deforming the xz trajectory and reducing minimum distance among platforms





From the equations, it is easy to calculate which perigee precession can be accepted based on maximum acceptable variation for upper limit of effective baseline, given by

$$a\sqrt{\delta e^2 \sin^2\vartheta + (\delta\Omega \sin i \cos\vartheta)^2 - 2\delta e \sin\vartheta\, \delta\Omega \sin i \cos\vartheta \cos\omega}$$

Assuming for example a reduction to 500 m, it is possible to derive ω = 60.7°

Conclusions

- A design-oriented analytical model was presented, which allows a time-explicit representation of relative motion between two satellites on the basis of orbital parameters differences.
- Second order terms allow to understand dynamics in large formations, whereas a linear approximation can be used in the close formation case.
- Two sets of equations were derived, considering the cases of circular or slightly eccentric reference orbit, both accounting for secular J_2 effects.
- Numerical simulations showed that modeling error is in the order of 0.1% on long timescales (hundreds of orbits) and thus can be considered acceptable for design requirements.
- A formation design example was briefly described regarding a cross-track INSAR mission. Relative trajectory was optimized on the basis of geometry requirements and possibilities/risks deriving from natural dynamics were pointed out.