Utilizing Evolutionary Algorithms for Problems with Vast Infeasible Regions and Expensive Function Evaluations

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Outline

- Evolutionary Algorithm Overview
- GTOC3 Description
- Overview of GTOC3 Difficulties
- Grid Search Mitigation
- Inner Loop Mitigation
- Multi Fidelity Search
- Future Research
- Conclusions

Evolutionary Algorithm (GRIPS)

- Aerospace Corporation tool based on Deb's NSGA2 algorithm
  - Limitations of NSGA2
    - Random seed variability of solutions
    - Potential for long duration run times
    - Trial and error analysis for parameterization
  - GRIPS improvements to NSGA2
    - Epsilon dominance archiving allows the user to control solution precision
    - Auto-adaptive population sizing helps prevent local convergence
    - Time continuation reinvigorates the search when it stagnates
    - Parallelized using the master-slave model
    - Utilizes method of generational synchronization that insures nearly 100% processor utilization
  - API (no existing trajectory optimization capability)

GTOC 3 Description

- Near Earth Asteroid rendezvous tour
  - Mission begins with hyberbolic excess velocity relative to Earth
  - Must rendezvous with 3 out of 140 asteroids for at least 60 days each
  - Mission ends with Earth rendezvous
  - 10 year launch window
  - 10 year mission duration
- System model
  - Initial mass 2000 kg, Thrust=0.15 N, Isp = 3000 s
  - Asteroids and Earth in Keplarian orbits about Sun
  - Central body gravity only
    - Patched conic flybys of Earth allowed at any time during mission

GTOC 3 Difficulties

- High fidelity search of entire space is computational intractable
  - 2.5 million asteroid tour combinations
  - Numerous permutations of gravity assists
  - Long mission duration and large launch window
  - Solution due one month after problem announcement
    - Much of the simulation code must be written or modified for use with GTOC3
- Vast regions of the search space are infeasible
  - For many selections of event dates (asteroid departure/arrival, etc), engine is not capable of completing the transfer
  - Including selection of thrust parameters causes nearly all solutions to fail to meet rendezvous conditions

GTOC3 Optimization Problem

- Cost function $$J = \frac{m_f}{m_i} + 0.02\min\left(\tau_j\right)$$
- Decision variables
  - Discrete variables (Lambert Grid Search)
    - Asteroid sequence
    - Flyby sequence
  - Continuous variables (GRIPS)
    - Hyberbolic excess velocity
    - Event dates
    - Flyby parameters
  - Other variables (Inner Loop Optimizer)
    - Optimal spacecraft thrust throughout trajectory

Lambert Grid Search

- Flybys only allowed between asteroid encounters
- Perform a Lambert solution for each possible transfer
  - 1 day grid for earth to asteroid or asteroid to asteroid transfers
  - 10 day grid for asteroid to flyby to asteroid transfers
  - Retain the best single leg solution for 0-1 rev, 1-2 revs, and 2-3 revs
- Examine all permutations of best single leg solutions (>500 million) to construct complete impulsive solutions
  - Use branch and bound to eliminate permutations where total Δv exceeds 8 km/s or where constraints are violated
- Technique is well suited to parallel processing
  - Performed over 5 trillion permutations
  - Computational time required: 768 days

Candidate Cases

	Leg #1					Leg #2							Leg #3							Leg #4						
Total Delta V	Departure JD	Departure Body	Arrival Body	TOF to Arrival Body	# Revs to Arrival Body	Departure JD	Departure Body	Arrival Body	Time to Earth Flyby	# Revs to Earth Flyby	TOF to Arrival Body	# Revs to Arrival Body	Departure JD	Departure Body	Arrival Body	Time to Earth Flyby	# Revs to Earth Flyby	TOF to Arrival Body	# Revs to Arrival Body	Departure JD	Departure Body	Arrival Body	TOF to Earth	# Revs to Earth	Total TOF	Total Delta V
5.9363	58376	0	76	904	2	59478	76	88	0	0	212	0	59798	88	49	0	0	200	0	60698	49	0	905	2	3227	5.9363
6.0341	58092	0	88	999	2	59778	88	19	0	0	504	1	60428	19	49	0	0	203	0	60698	49	0	905	2	3511	6.0341
6.1982	58450	0	96	254	0	59188	96	88	0	0	327	0	59588	88	49	0	0	584	1	60698	49	0	905	2	3153	6.1982
6.2029	59189	0	88	1100	2	60948	88	11	0	0	887	2	61968	11	49	0	0	281	0	62318	49	0	500	1	3629	6.2029
5.3918	59401	0	96	691	1	60318	96	76	0	0	579	1	60958	76	49	920	2	230	0	62318	49	0	594	1	3511	5.3918
5.3948	58463	0	88	1100	2	59778	88	19	0	0	504	1	60418	19	49	520	1	790	2	61788	49	0	200	0	3525	5.3948
5.4407	58464	0	88	731	1	59458	88	76	0	0	252	0	59848	76	49	950	2	710	1	61768	49	0	204	0	3508	5.4407
5.6549	58767	0	37	831	2	59998	37	85	0	0	477	1	60568	85	49	910	2	250	0	61788	49	0	200	0	3221	5.6549
5.7477	58104	0	88	921	2	59088	88	11	0	0	237	0	59418	11	49	290	0	800	2	60698	49	0	905	2	3499	5.7477
4.8755	60720	0	49	904	2	62118	49	37	650	1	200	0	63128	37	85	0	0	263	0	63848	85	0	293	0	3421	4.8755
4.9672	58084	0	88	209	0	58418	88	96	590	1	300	0	59468	96	49	810	2	680	1	61038	49	0	577	1	3531	4.9672
5.3369	58449	0	96	255	0	58888	96	19	230	0	930	3	60418	19	49	520	1	790	2	61788	49	0	200	0	3539	5.3369
5.3787	58450	0	96	254	0	58878	96	37	260	0	1040	3	60308	37	49	990	3	410	1	61768	49	0	204	0	3522	5.3787
5.4160	58450	0	96	254	0	58858	96	30	270	0	1090	3	60488	30	49	520	1	480	1	61768	49	0	204	0	3522	5.4160
5.4360	58450	0	96	254	0	58798	96	111	870	2	700	1	60448	111	49	670	1	610	1	61788	49	0	200	0	3538	5.4360
5.5544	58447	0	96	257	0	58848	96	64	290	0	920	2	60228	64	49	940	2	360	0	61768	49	0	204	0	3525	5.5544
5.6003	60968	0	49	998	2	62118	49	37	650	1	200	0	63028	37	85	250	0	650	1	64208	85	0	286	0	3526	5.6003

Inner Loop Optimization

- Allow the outer loop to provide the sequence including event dates
  - Initial states, final states, and transfer time are inputs from outer loop
  - Most selections of thrust direction and magnitude will yield infeasible solution
- Optimization problem is reduced to determining a series of optimal intercept (flyby) or rendezvous fixed time transfers
  - Find optimal thrust pointing history and thrust durations to achieve transfer with minimum fuel
  - Optimizer is ideally closed-loop: starts with a generic initial guess and converges with no user intervention
    - Allows local optimizer to be plugged into outer optimizer (EA) which adjusts the input parameters sent to the local optimizer (Initial and final times and states)

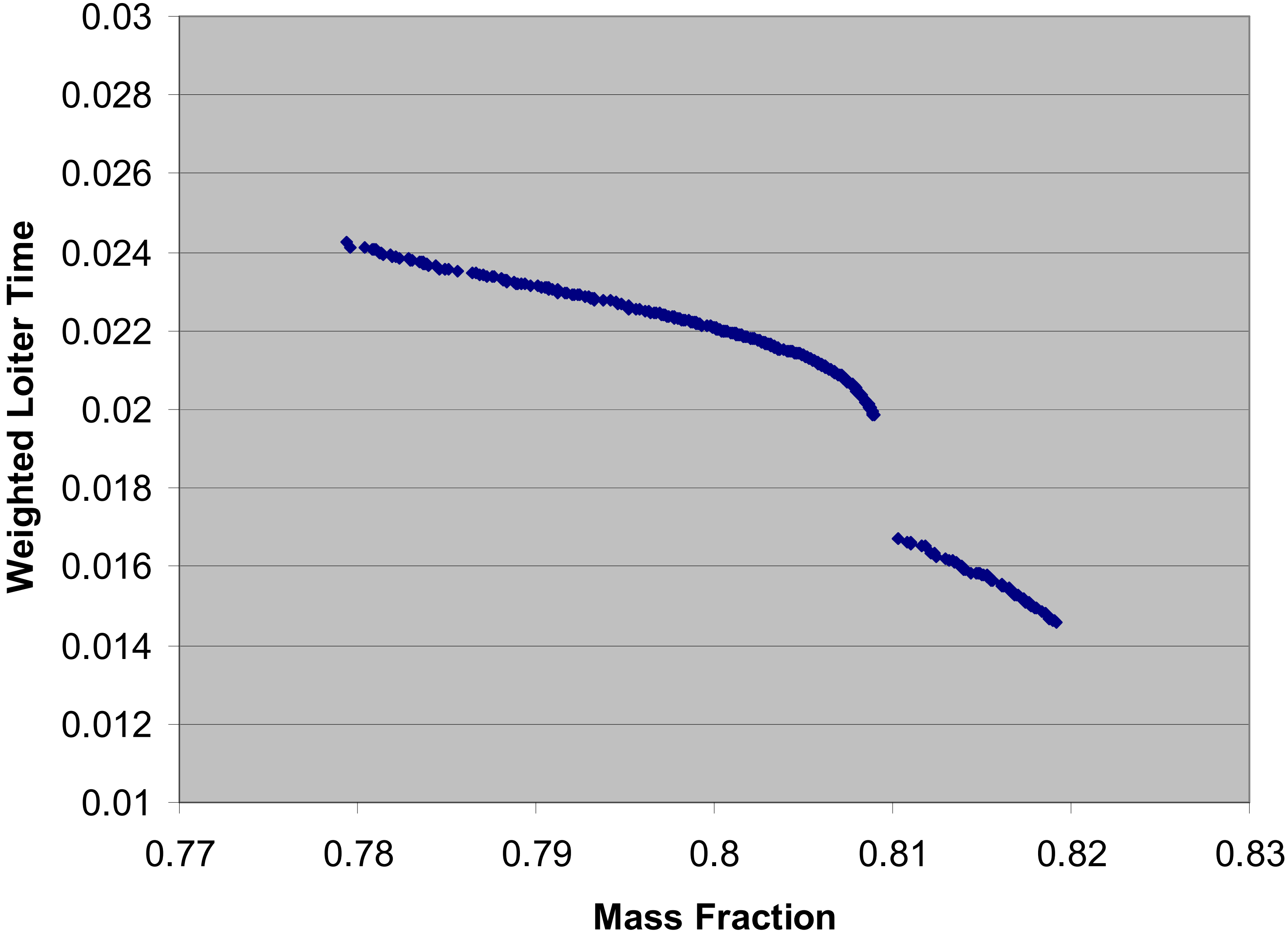
Inner Loop Technique

- Indirect optimization technique utilizing adjoint control transformation
  - Two-point boundary value problem: equal number of initial unknowns (adjoints and problem parameters) and final optimality and physical problem constraints
  - Cartesian coordinates used for this application due to time constraints on solution
  - Differential equations describe infinite dimensional optimal control history
  - Solved with FORTRAN subroutine using a boundary value solver from Harwell Subroutine Library Archives (Broyden's Method combined with Steepest Descent)
- Difficult to find generic initial guess that converges for all feasible transfers
  - Start with a simpler problem and morph it into the actual problem dynamics
    - Initial thrust set to 10*Actual Thrust (1.5 N)
    - Shorter TOF for initial trajectory of 270 days
- Incrementally increase TOF to desired value while decreasing thrust after converging each simplified trajectory
  - If simplified trajectories do not converge, transfer is infeasible and inner loop returns an error message to the EA

EA Decision Variable Ranges

Asteroid Sequence	88-96-49
Earth Departure Date	57568 to 57968
Departure V∞	0 to 0.5
Departure RA	0 to 360
Departure Dec	0 to 180
TOF to Asteroid 1	187 to 587
Stay Time Asteroid 1	60 to 443
TOF to	436 to 836
Flyby Perigee Radius	6871 to 40000
Flyby Angle	0 to 360
TOF to Asteroid 2	187 to 587
Stay Time Asteroid 2	152 to 552
TOF to Asteroid 3	372 to 572
Stay Time Asteroid 3	113 to 513
TOF to Earth	494 to 894

Non-Dominated Front



GRIPS Optimal Values

Asteroid Sequence	88-96-49
Earth Departure Date	57738
Departure V∞	0.5
Departure RA	228
Departure Dec	70
TOF to Asteroid 1	377
Stay Time Asteroid 1	267
TOF to Flyby	627
Flyby Perigee Radius	6871
Flyby Angle	331
TOF to Asteroid 2	362
Stay Time Asteroid 2	425
TOF to Asteroid 3	492
Stay Time Asteroid 3	353
TOF to Earth	676
Objective Value	0.83345

Low Fidelity to High Fidelity Transition Problems

- It is not feasible to perform all simulations in high fidelity
- Low fidelity search may eliminate optimal high fidelity solutions
  - Not allowing flybys before the first gravity assist eliminated best solutions
  - Mapping from low fidelity to high fidelity is not perfect
    - If mapping is perfect, high fidelity is unneeded
- Feedback from high fidelity simulations is needed for low fidelity decision making

Mixed Fidelity Search

- First perform a low fidelity simulation on a candidate
  - Quickly determine if candidate is near the optimal
  - Low fidelity search can be approximate
    - Exact ranking of these solutions in unnecessary
  - Goal is to save as many high fidelity searches as possible
- Examine near optimal solutions in higher fidelity
  - Provide an exact ranking for near optimal solutions
  - Desire to perform a high fidelity simulation for any low fidelity solution that could be optimal in high fidelity
    - The number of high fidelity simulations is dependent on the accuracy of the low fidelity simulation
      - Low fidelity simulation will trade accuracy for simulation time

Multi Fidelity Example

- MOP2 test problem from Huband et. al.
  - Separable, unimodal minimization problem where the solution is not at the extremal or medial values of the decision variables
- Variables range from -4 to 4
- High fidelity simulation evaluates function using double precision
- Low fidelity simulation rounds each decision variable to nearest 0.1 and then perturbs the decision variable by a value between -0.1 and 0.1

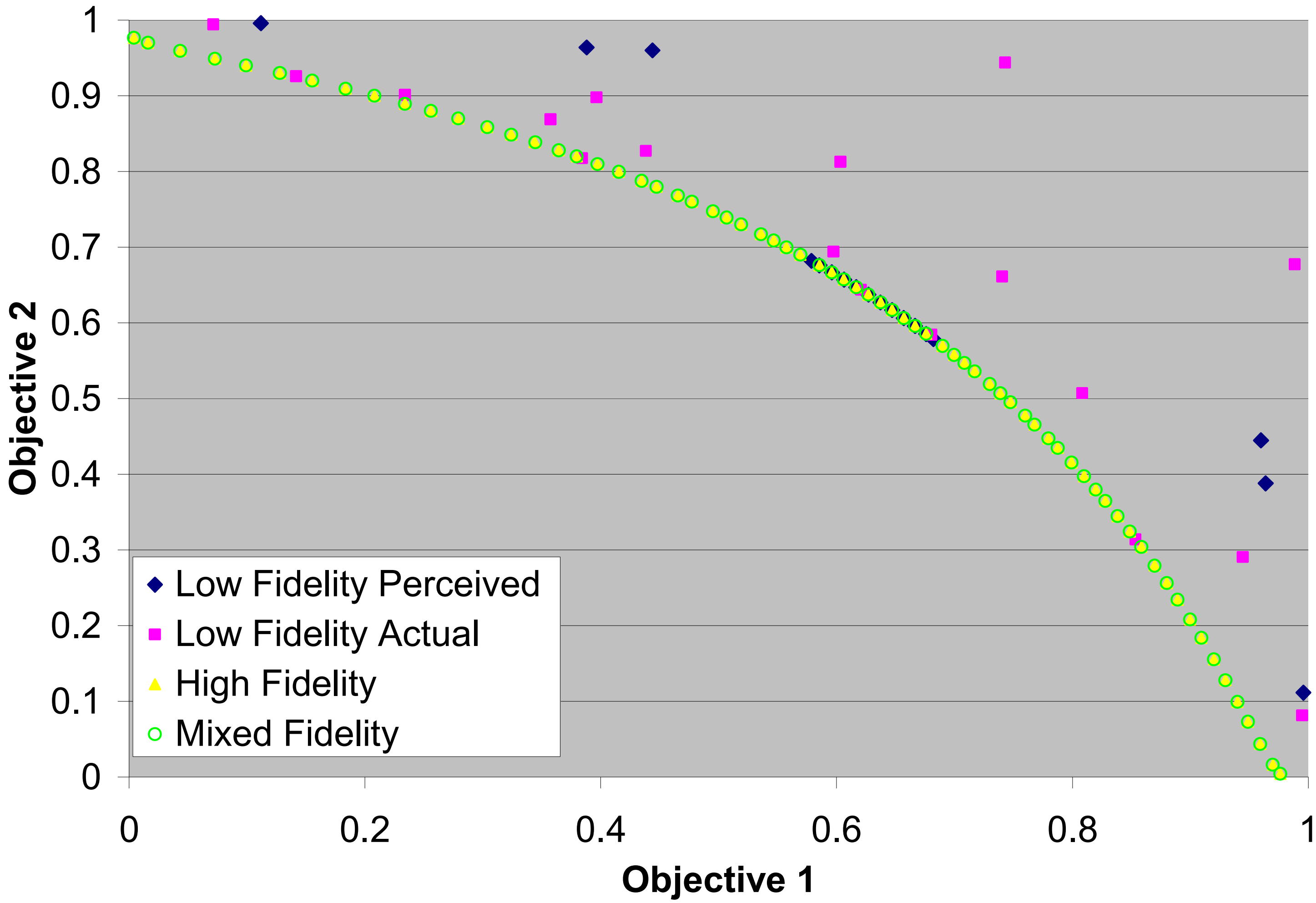
$$f_1 = 1 - \exp\left[-\left(x_1 - \frac{1}{\sqrt{2}} \right)^2 - \left(x_2 - \frac{1}{\sqrt{2}} \right)^2 \right]$$

$$f_2 = 1 - \exp\left[-\left(x_1 + \frac{1}{\sqrt{2}} \right)^2 - \left(x_2 + \frac{1}{\sqrt{2}} \right)^2 \right]$$

Solution Technique

- Need all high fidelity objective values to be more optimal than low fidelity objectives
  - Add one to the any low fidelity objective
- Determine when to perform a high fidelity simulation
  - Utilize a brief run completely in low fidelity and determine the non-dominated front
  - Fit a curve to the front (quadratic)
  - Any low fidelity simulation that is better than the curve is performed in high fidelity

Objective Space Results



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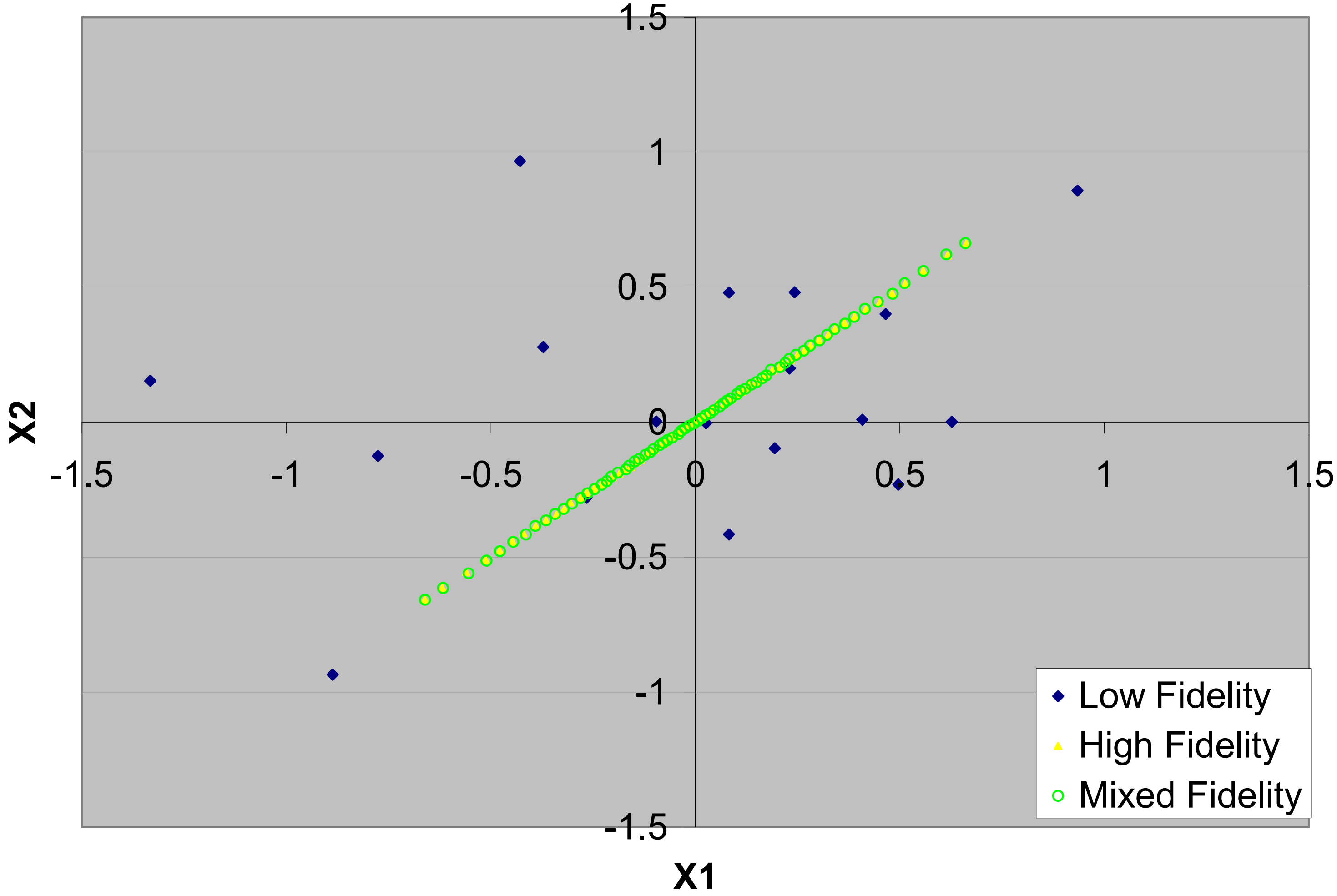
Decision Variable Results



Multi Fidelity Future Work

- Determining when to perform high fidelity simulations
  - Static decisions
    - Top percentage of low fidelity simulations
    - Low fidelity simulation better than pre determined value
  - Dynamic decisions
    - Maintain an evolving low fidelity front
    - Periodically examine all solutions in high fidelity
      - Determine appropriate percentage to examine in high fidelity for future generations
- Efficient parallel computer utilization
  - Low fidelity solutions may run orders of magnitude more quickly
    - Low fidelity simulations will achieve numerous generations during a single high fidelity simulation
    - Fast low fidelity simulations may require packets of chromosomes be passed to a single computer node

Conclusions

- Evolutionary algorithm successfully selected event dates and flyby parameters
  - Inner loop indirect optimization routine operated robustly and autonomously
- Initial assumptions and limited success in mapping from low fidelity to high fidelity solutions eliminated many of the optimal solutions
- Mixed fidelity simulation offers the potential to examine more of the space without falsely eliminating as many potential solutions
- Significant work is needed to efficiently utilize mixed fidelity simulations for real world problems

Questions